5.1 Perpendiculars and Bisectors

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Goals
- Use properties of perpendicular bisectors.
- Use properties of angle bisectors to identify equal distances.

VOCABULARY

Perpendicular bisector

Equidistant from two points

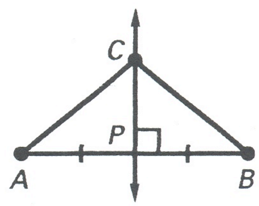
Distance from a point to a line

Equidistant from two lines

THEOREM 5.1: PERPENDICULAR BISECTOR THEOREM

If a point is on the perpendicular bisector of a segment, then it is equidistant from the __________ of the segment.

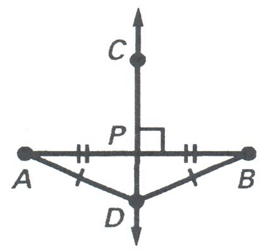
If $\overleftrightarrow{CP}$ is the perpendicular bisector of $\overline{AB}$, then ____ = ____.



THEOREM 5.2: CONVERSE OF THE PERPENDICULAR BISECTOR THEOREM

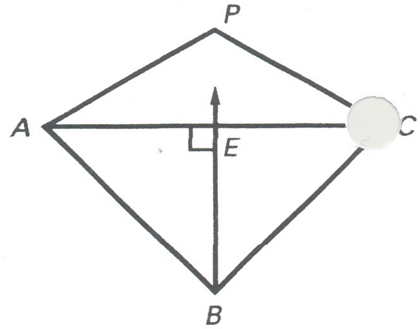
If a point is __________ from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

If ____ = ____, then D lies on the perpendicular bisector of $\overline{AB}$.



Example 1 Using Perpendicular Bisectors

In the diagram shown, $\overrightarrow{BE}$ is the perpendicular bisector of $\overline{AC}$.



a. What segment lengths are equal?

b. $\overline{AP} \cong \overline{CP}$. What can you conclude about point P?

Solution

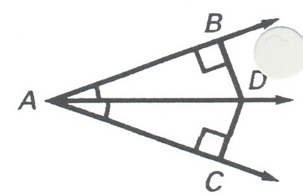
a. Because $\overrightarrow{BE}$ bisects $\overline{AC}$, ____ = ____.

Because B is on the perpendicular bisector of $\overline{AC}$, you can use the ________________ Theorem to conclude that ____ = ____.

b. Because $\overline{AP} \cong \overline{CP}$, $AP =$ ____. Using the ____________ ________________ Theorem, you can conclude that __________.

THEOREM 5.3: ANGLE BISECTOR THEOREM

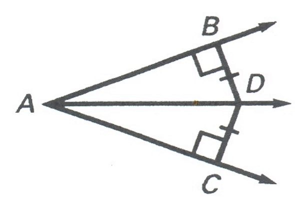
If a point is on the bisector of an angle, then it is equidistant from the two ______ of the angle.



If $m\angle$_____ $= m\angle$_____, then $DB = DC$.

THEOREM 5.4: CONVERSE OF THE ANGLE BISECTOR THEOREM

If a point is in the interior of an angle and is equidistant from the ______ of the angle, then it lies on the bisector of the angle.



If ____ = ____, then $m\angle BAD = m\angle CAD$.

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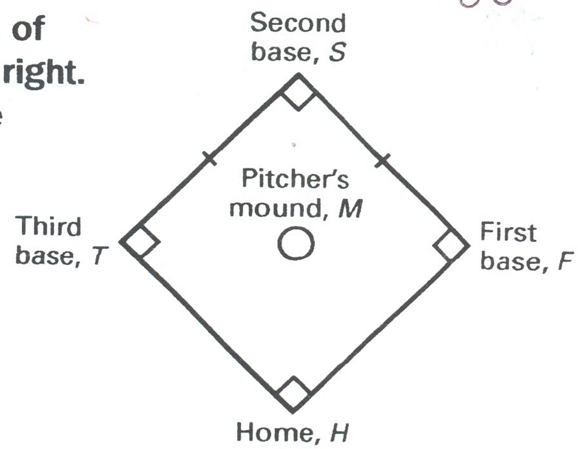
Example 2 Using Angle Bisectors

Baseball Field Use the diagram of the baseball infield shown at the right. What can you conclude about the measure of $\angle SHF$?



Solution

From the diagram, you know that point ___ is in the interior of $\angle THF$ and $ST =$ ___.

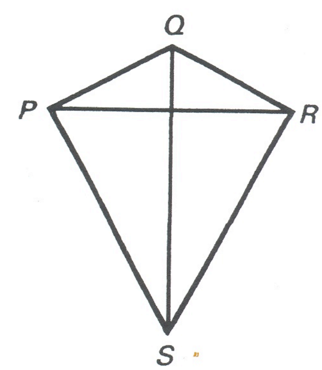
From the ________________ ________ Theorem, you know that S lies on the angle bisector of $\angle$_____. An angle bisector divides an angle into two congruent angles, each of which has _____ the measure of the original angle, so

$$m\angle SHF = \frac{\qquad}{\_\_\_\_} = \_\_\_^\circ.$$

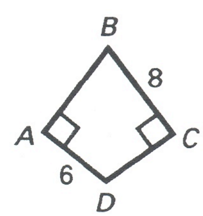
Answer The measure of $\angle SHF$ is ___°.

Checkpoint Complete the following exercises.

1. In the diagram, $\overline{PQ} \cong \overline{RQ}$. What conclusion can you make about point Q? Can you conclude that S is on the perpendicular bisector of $\overline{PR}$? Explain.



2. In the diagram, D is on the bisector of $\angle ABC$. What is DC? Explain.



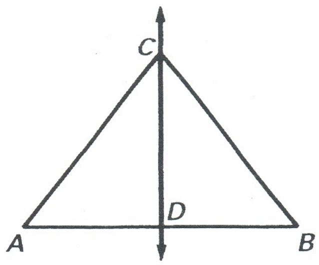
LESSON 5.1

NAME ______________________ DATE __________

Practice A

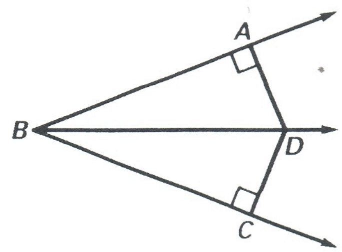
For use with pages 264–271

Use the diagram shown. $\overleftrightarrow{CD}$ is the perpendicular bisector of $\overline{AB}$.



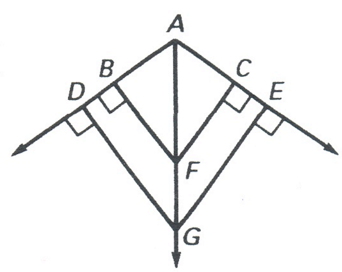
1. What is the relationship between AD and AB?
2. What is the relationship between $\angle ADC$ and $\angle BDC$?
3. What is the relationship between AC and CB? Explain.
4. ***True or False?*** Because $\overleftrightarrow{CD}$ is the perpendicular bisector of $\overline{AB}$, $\overline{AC} \cong \overline{AD}$.

Use the diagram shown. $\overrightarrow{BD}$ is the angle bisector of $\angle ABC$.



5. What is the relationship between $\angle ABD$ and $\angle CBD$?
6. What is the relationship between $\angle DAB$ and $\angle DCB$?
7. What is the relationship between AD and CD? Explain.
8. ***True or False?*** Because $\overrightarrow{BD}$ is the angle bisector of $\angle ABC$, $\overline{AB} \cong \overline{CB}$.

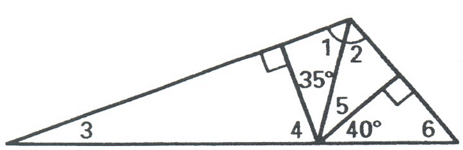
Use the diagram to answer the following.



9. If $m\angle BAF = {}^\circ 54$, then $m\angle CAF = \underline{\ ?\ }$.
10. If $FC = 16$, then $FB = \underline{\ ?\ }$.
11. If $\overline{GD} \cong \overline{GE}$, then what can you conclude about point G?
12. Is $\triangle ABF \cong \triangle ACF$? Explain.

Find the measure of the numbered angles.

13.



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